

How Long Does It Take a Person to Sober Up? Some Mathematics and Science of DUI

The number of annual accidents and fatalities related to driving under the influence (DUI) across the United States is staggering. In 2008 alone, drivers under the influence caused 11,773 deaths, representing 32% of all highway fatalities that year. That is an average of one DUI-related death every 45 minutes, a number greater than three times the number of people killed on 9/11 and equivalent to the number of people killed if a large passenger airplane were to crash every week.

In most secondary schools, the task of preparing students to make good, educated decisions about drinking and driving is left almost solely to driver education and health instructors. By spending more time investigating the topic and using pertinent mathematics and science concepts, mathematics teachers too can help students develop a greater understanding of the risk involved when people drink and drive.

This activity provides an interdisciplinary approach to teaching mathematics in grades 9–11, integrating real-world mathematics across such curricular areas as health education, biology, physics, chemistry, social studies, and driver education. The entire DUI teaching module includes ten investigations, each focusing independently on a different aspect of the DUI problem. DUI accident statistics, absorption and elimination rates of alcohol, car

"Activities for Students" appears six times each year in *Mathematics Teacher*, often providing in reproducible formats activity sheets that teachers can adapt for use in their own classroom. Manuscripts for the department should be submitted via <http://mt.msubmit.net>. For more information on the department and guidelines for submitting a manuscript, please visit <http://www.nctm.org/publications/content.aspx?id=10440#activities>.

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crashes, laws and law enforcement, and ensuing financial ramifications of DUI-related accidents and fatalities provide an opportunity to introduce or review important mathematics concepts.

This investigation illustrates the elimination of alcohol from the body. The results provide some surprising information for drivers of any age and driving experience.

Over the course of one to three days, this investigation will familiarize students with zero-order drugs (e.g., alcohol) and first-order drugs (e.g., caffeine and penicillin) and the difference between their elimination rates from the human body. Students must know how to plot points and how to interpolate and extrapolate data from both linear and exponential functions as well as an understanding of graphing inequalities.

Using data tables for caffeine and alcohol elimination, students will graph linear and exponential functions, determine best-fit equations, understand what the constant quantities in those equations represent, and graph linear inequalities. Finally, students will understand the concept of half-life and be able to describe the difference between zero-order and first-order drugs. The use of graphing calculators (TI-83 or similar) is assumed.

INTRODUCING THE INVESTIGATION

Alcohol is the most widely used central nervous system (CNS) depressant in the world. The most widely used CNS stimulant in the world, in contrast, is caffeine. Although alcohol is a controlled substance in most countries, caffeine is available to anyone who wants it. Both chemicals pass through the stomach and are then absorbed through the small intestine and into the bloodstream. They are later excreted through urine and other body fluids after passing through the liver and kidneys. But does the human body process them in similar ways?

Begin this investigation with a general discussion of decay—that is, the diminishing of some substance in a system over time. Decay is often described by linear and exponential functions. If the decay is exponential, then the amount of the substance present divides at a constant time interval called a half-life.

To acquaint students better with these concepts, we present a short case study on penicillin. Most students have taken antibiotics, so they can easily relate to this application. We ask students to make conjectures from the data in **table 1**.

Classroom discussion of the table produced the following comments from students:

- “L1 could be time.”
- “L2 is probably the amount of penicillin.”
- “After half an hour, half of the penicillin is gone.”

Table 1 The Case of Penicillin

L1	L2
0	600,000
0.5	300,000
1.0	150,000
1.5	75,000
2.0	37,500
2.5	18,750
3.0	9,375
3.5	4,688
4.0	602,344

- “After one hour, three-fourths of the penicillin is gone.”
- “After four hours the person takes another dose.”

After this introduction, we ask students to sketch a graph that they think illustrates what happens to the drug in the first 12 hours after it has been ingested. We then ask students to share their graphs with one another, and, because it is unlikely that each student has made the same predictions, a spirited discussion usually ensues. Finally, we show how the graphs would look had the students used their graphing calculators, or we ask students to obtain a graph after linking the data to their calculators.

ALCOHOL ABSORPTION AND ELIMINATION

The human body begins to process alcohol as soon as it is ingested. Although some alcohol is absorbed into the bloodstream from the mouth, the vast majority passes into the stomach. If the stomach is full, most of the alcohol is absorbed into the surrounding food, which passes slowly through the pyloric sphincter, or valve, into the small intestine. As the food is digested, the alcohol passes into the bloodstream through the walls of the small intestine.

If the stomach is empty, however, the pyloric valve remains open. In this case, more than 80% of the alcohol goes straight into the small intestine, and as much as 20% passes into the bloodstream through the stomach lining, because there is no food to impede it. For this reason, alcohol absorption on an empty stomach is much more rapid than on a full stomach.

Once in the bloodstream, alcohol travels quickly throughout the body, permeating most body tissues in a matter of minutes or even seconds. About 90% of the absorbed alcohol is metabolized in the liver, where it eventually breaks down into carbon dioxide and water. About half the remaining 10% is excreted in fluids, such as sweat and urine, and the rest diffuses into the lung, from which it is exhaled.

In this investigation and the accompanying activity sheets, students will compare the elimination rates of caffeine and alcohol from the human body. In comparing these processes, they will learn about zero- and first-order drugs, exponential decay, half-life, and linear decay.

OUR REFLECTIONS

Teachers who include the topic of driving under the influence of alcohol as part of a first-year algebra course will not only teach applied mathematics. They will also help students understand that, although alcohol is a legal substance for adults, all persons, regardless of age, need to learn how to protect themselves and others from the often-tragic consequences when people drive while intoxicated.

The topic of DUI is of high interest to our students because they are just beginning to drive. As a result of this investigation, they seem to become much more aware of their own behavior and also more careful about whom they drive with. It is crucial that students find their own data, especially in investigations that deal with geographical and demographic differences. Students are always curious and motivated by their findings—and that is a good thing.

SOLUTIONS

Caffeine Elimination Rate (Sheet 1)

1. The scatter plot should look similar to the one shown in **figure 1**.
2. Answers will vary but should describe the exponential decay of caffeine.
3. The equation of the best-fit curve to this graph is $y = 48 \cdot 0.794^x$.

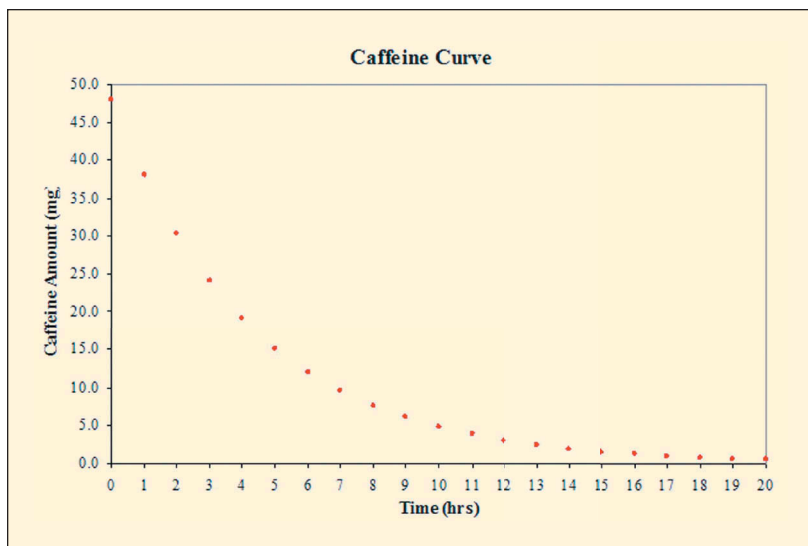


Fig. 1 The scatter plot of the data (activity sheet 1, question 1) resembles this graph.

4. In this equation, a is the starting caffeine amount (in mg), and b is the fractional reduction of the caffeine amount per unit of time (mg/hr).
5. In any system that exhibits exponential decay, the term *half-life* describes the time it takes for the material present to diminish by half. According to the data in question 1, the half-life of caffeine in the body is 3 hours. (Actually, the half-life of caffeine varies widely in humans because age, pregnancy, smoking, and other factors influence it. As with alcohol, caffeine is more rapidly absorbed on an empty stomach than a full one and for the same reasons. For the average adult, the half-life of caffeine is 3–6 hours.)
6. The mathematics shows that the caffeine will never be completely eliminated because the function never reaches a zero value. At some point, however, the function will reach values that can describe only fractional parts of caffeine molecules, beyond which the substance can no longer accurately be called caffeine. Also remember that the half-life model is statistical, not deterministic—the smaller the number of caffeine molecules, the less accurate the model is. Some molecules may stay in the system indefinitely, but they may also be completely eliminated at some point.

Alcohol Elimination Rate (Sheet 2)

1. The scatter plot should look similar to the one shown in **figure 2**.
2. The equation for the best-fit line to this graph is $y = -0.015x + 0.19$.
3. In this equation, a represents the amount the blood alcohol concentration (BAC) level drops every hour (grams/deciliter), and b is the starting level of BAC in g/dl.
4. Students can solve this equation algebraically or by using TRACE on their graphing calculators:

$$0.08 = -0.015x + 0.19$$

$$-0.11 = -0.015x$$

$$x = 7\frac{1}{3} \text{ hrs}$$

According to the model, Bryan would not be able to drive legally until after 7:20 the next morning.

5. Again, students can solve this equation algebraically or by using TRACE:

$$0 = -0.015x + 0.19$$

$$-0.19 = -0.015x$$

$$x = 12\frac{2}{3} \text{ hrs}$$

According to the model, Bryan will not be totally sober until 12:40 the next afternoon.

6. The inequalities are $y > 0.08$ and $y \leq -0.15x + 0.19$.
7. Any level of alcohol in the body can impair driving judgment. For this reason, zero tolerance for teenagers is the law in some states. Some countries have a legal limit as low as 0.02 g/dl.
8. At 0.01 g/dl per hour, Bryan will not be completely sober for 19 hours, or until 7:00 the next evening.

As with caffeine, the exact rate of alcohol elimination from the human body varies. It usually ranges between 0.01 and 0.02 g/dl per hour, but it is affected by such factors as age, weight, gender, food consumption, and drinking frequency.

9. Answers will vary, but students should explain that the elimination pattern of zero-order drugs is linear whereas that of first-order drugs is exponential, so the latter have a half-life.
10. Students should provide an accurate summary of the difference between zero-order and first-order drugs as described by the elimination curves. They should also demonstrate a strong understanding of how long it takes for alcohol to be completely eliminated from the body.

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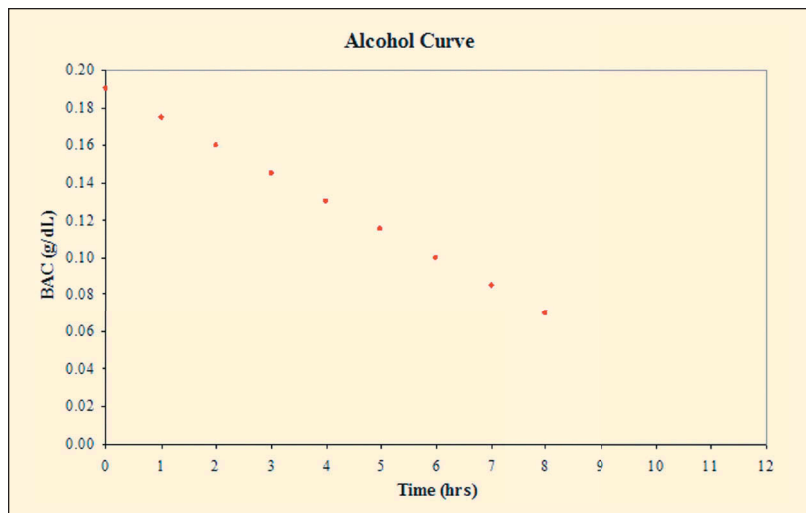


Fig. 2 A linear function models alcohol elimination (**activity sheet 2**, question 1).

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Caffeine Elimination Rate

Sheet 1

The human body processes different drugs in different ways. Most of you have probably observed that a drunken person does not stay drunk forever. Sooner or later, the blood alcohol concentration (BAC) returns to zero. Alcohol is a rather unusual chemical in terms of how the body processes it. To understand it better, let's compare it with the caffeine in an average soft drink.

Assume that you quickly drink a 12 oz. soft drink containing about 48 mg of caffeine. If the amount of caffeine in your body were monitored over the next 19 hours, the result would be similar to the data in the following table.

Caffeine Elimination Rate			
Time (hrs)	Caffeine (mg)	Time (hrs)	Caffeine (mg)
0	48.00	10	4.78
1	38.11	11	3.80
2	30.26	12	3.01
3	24.03	13	2.39
4	19.08	14	1.90
5	15.15	15	1.51
6	12.03	16	1.20
7	9.55	17	0.95
8	7.58	18	0.76
9	6.02	19	0.60

1. Make a scatter plot of the data (time, caffeine) from the table.
2. Examine the scatter plot and the table. How would you best describe the trend?
3. Determine the best-fit regression equation for the graph.
4. The general form of the equation for this graph is $y = ab^x$. Taking note of the quantities you have just graphed, explain the meaning of the constants a and b in this equation.
5. What does *half-life* mean? What is the half-life of caffeine in this particular scenario?
6. When will your body have eliminated all the caffeine?

Caffeine, like most common drugs, is classified as a first-order drug because exponential decay accurately models its elimination from the human body. Alcohol, on the other hand, is a zero-order drug. Let's look at what the difference is.

Alcohol Elimination Rate

Sheet 2

Assume that 24-year-old Bryan has been eating and drinking for most of the evening and by midnight has a blood alcohol concentration (BAC) level of 0.19 g/dl (grams/deciliter)—far above the legal driving limit of 0.08 g/dl. If Bryan weighs 140 lbs., it would take approximately seven drinks for his BAC to reach 0.19 g/dl. Fortunately for Bryan, a friend has already taken his car keys. The question now is, When should the keys be given back to Bryan? If someone were to monitor Bryan's BAC over the next 8 hours, it would most likely look like the data in the following table.

Alcohol Elimination Rate					
Time (hrs)	Alcohol (g/dl)	Time (hrs)	Alcohol (g/dl)	Time (hrs)	Alcohol (g/dl)
0	0.190	3	0.145	6	0.100
1	0.175	4	0.130	7	0.085
2	0.160	5	0.115	8	0.070

1. Make a scatter plot of the data (time, alcohol) from the table.
2. Determine the best-fit regression equation for this graph.
3. The general form of the equation for this graph is $y = ax + b$. In terms of the quantities you have just graphed, explain the meaning of the constants a and b in this equation.
4. As mentioned, Bryan wisely stopped drinking at midnight. Legally, Bryan cannot drive home until his BAC is at or below 0.08 g/dl. When will he be sober enough, legally, to drive home? Note that the 0.08 g/dl BAC level is statutory in all states; however, many states have “zero tolerance” laws for persons under the age of 21. Under these laws, drivers who are not yet 21 cannot have *any* alcohol in their body while driving.
5. When was all the alcohol eliminated from Bryan's body?
6. On the calculator or on paper, shade the region that shows the no-drive zone for Bryan. Write the two inequalities that describe this region.
7. Does the term *legal drive zone* imply that this zone is necessarily a safe drive zone?
8. The elimination rate of alcohol from the human body is highly variable. The rate you found here is an average value, but elimination rates as low as 0.01 g/dl per hour are common. At this conservative rate, at what time would the alcohol have been eliminated from Bryan's body?
9. What are the differences between zero-order drugs (such as alcohol) and first-order drugs (such as caffeine)?
10. Write a short reflection on this investigation, summarizing what you have learned as well as any personal reactions.